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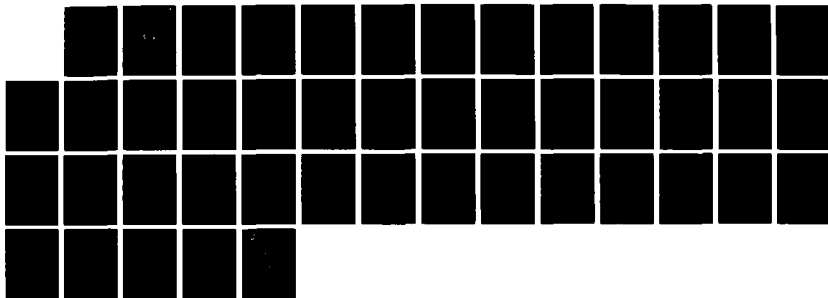
A METHOD FOR ONLINE TESTING BY HOC (HIGHER ORDER  
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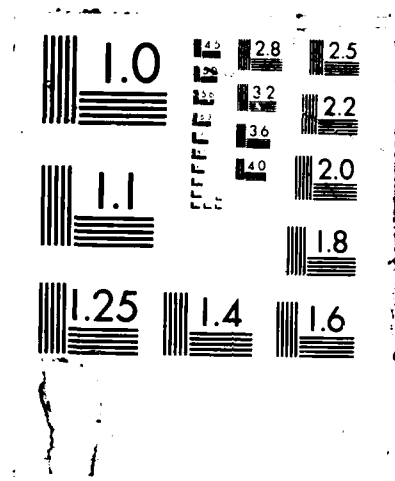
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A METHOD FOR ONLINE TESTING BY HOC-PROCESSES

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### Abstract

The dynamics by which a stationary time series produces its zero-crossings and higher order zero-crossings sequentially in time is studied, illustrated, and applied in white noise testing.

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## OUTLINE

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## 1. Introduction

Consider a stationary time series evolving in time about level zero. As the time series threads its sample path about this level the zero-crossing rate (the number of zero-crossings per unit time) converges in some sense as the series length increases. Similarly, the zero-crossing rate in the first difference of the series converges too, and the same can be said about the zero-crossing rates of higher order differences of the time series. In this paper we examine the dynamic process by which a time series and its higher order differences produce their respective zero-crossing counts sequentially in time for the purpose of white noise testing. Because a zero-crossing rate as a function of the series length is a stochastic process, we can reiterate by saying that we shall study zero-crossing rate processes obtained from the original time series by repeated differencing. Such processes will be called higher order crossings rate processes or simply HOC-processes. The present work is an extension of the methods and ideas discussed in Kedem (1987) and in Kedem and Martin (1987) where HOC plots and HOC-processes are introduced as graphical tools in time series model identification. Here the emphasis is on a graphical tool that is more powerful than HOC plots for white noise testing.

There is a difference between regular higher order crossings (HOC) and HOC-processes. HOC are zero-crossing counts in filtered time series where the filtering may consist of any combination of linear filters applied sequentially or repeatedly. The distinction lies in the sample size. HOC are defined for fixed record

lengths while in HOC-processes the record lengths increase. In this paper by HOC we always mean higher order crossings obtained by repeated differencing. General properties of HOC are reviewed in Kedem (1986).

In general HOC constitute a monotone increasing sequence. This in turns implies that the  $(k+1)$ 'th order HOC-process tends to have sample paths that lie above the sample paths of the  $k$ 'th order HOC-process, a fact which induces a certain weak convergence of HOC-processes. Thus there are two basic modes of convergence associated with HOC-processes, "horizontal" that depends on the series size and "vertical" that depends on the difference order. Both modes of convergence summarize the oscillatory behavior of a time series. The initial rate of the vertical convergence is rather fast which means that very few HOC-processes are needed for our purpose.

The first part of the paper deals with basic properties of HOC and HOC-processes. Section 4 discusses in some detail the difficulty in obtaining the exact variance of HOC and suggests instead a useful approximation. Section 5 is where we describe and illustrate our test procedure.

## 2.0 DEFINITION OF HOC PROCESSES

We are given a time series  $\{Z_t\}$ ,  $t = \dots, -1, 0, 1, \dots$  with covariance function

$$\gamma(t, r) = \text{Cov}(Z_t, Z_{t-r}), \quad r, t = 0, \pm 1, \pm 2, \dots$$

and correlation function

$$\rho(t, r) = \gamma(t, r) / \gamma(t, 0), \quad r, t = 0, \pm 1, \pm 2, \dots$$

We assume the process is stationary with zero mean, so that

$$0 = E Z_t = E Z_{t-j} = E Z_{t+j}$$

$$\gamma(t, r) = E (Z_t Z_{t-r}) = \gamma_r$$

$$\rho_r = \gamma_r / \gamma_0,$$

$r = 0, \pm 1, \pm 2, \dots$ . Finally we assume that the process  $\{Z_t\}$  is Gaussian.

Let  $\{Z_t^{(0)}\} \equiv \{Z_t\}$  be the 0<sup>th</sup>-order difference process. To form the higher order difference processes, we begin by defining the first order difference process  $\{Z_t^{(1)}\}$  such that

$$Z_t^{(1)} = Z_t^{(0)} - Z_{t-1}^{(0)} \equiv \nabla Z_t.$$

In general for the  $k^{\text{th}}$ -order differences, define

$$Z_t^{(k+1)} = Z_t^{(k)} - Z_{t-1}^{(k)} = \nabla^{(k+1)} Z_t, \quad k = 0, 1, 2, \dots \quad (2.1)$$

The processes  $\{Z_t^{(k)}\}$  are referred to in the remainder of this paper as "Z-processes".

From the  $k-1^{\text{th}}$ -order difference process  $Z_t^{(k-1)}$  we define the  $k^{\text{th}}$ -order "clipped" process  $\{X_t^{(k)}\}$  such that

$$X_t^{(k)} = I(Z_t^{(k-1)} > 0), \quad t = 0, \pm 1, \pm 2, \dots \quad (2.2)$$



where  $I(\cdot)$  is the indicator function. The binary-valued processes  $\{X_t^{(k)}\}$  are referred to as "X-processes". We further define

$$d_t^{(k)} = I \left[ X_t^{(k)} \neq X_{t-1}^{(k)} \right] . \quad (2.3)$$

Thus  $d_t^{(k)} = 1$  whenever a zero-crossing occurs between  $Z_{t-1}^{(k-1)}$  and  $Z_t^{(k-1)}$ , and is 0 otherwise.

The total number of zero-crossings which occur between  $t = 1$  and  $t = N$  in the  $(k-1)^{\text{th}}$ -order process is then

$$D_{k,N} = d_2^k + d_3^k + \dots + d_N^k . \quad (2.4)$$

The processes  $\{D_{k,N}/N\}$  are referred to as HOC-processes or simply "D-processes", with  $k$  being the order and  $N, N \geq 2$ , the number of observations. Thus the first order HOC-process is  $\{D_{1,N}/N\}$ ,  $N = 2, 3, \dots$ . More precisely,  $\{D_{k,N}/N\}$  is a sequence of processes with parameter  $N$  indexed by  $k$ .

### 3.0 PROPERTIES OF D-PROCESSES

#### 3.1 ACF relationships

Because the process  $\{Z_t^{(k+1)}\}$  is obtained from simple differencing of the process  $\{Z_t^{(k)}\}$ , there is a direct relationship between the correlation functions of the Z-processes.

Let

$$\gamma_t^{(k)} = \text{covariance function for } \{Z_t^{(k)}\}$$

$$\rho_t^{(k)} = \text{correlation function for } \{Z_t^{(k)}\}.$$

From the definition of  $\{Z_t^{(k)}\}$ ,

$$Z_t^{(k)} = \sum_{i=0}^k (-1)^i \binom{k}{i} Z_{t-i} \quad k = 0, 1, \dots \quad (3-1)$$

so that

$$\gamma_r^{(k)} = E \left\{ Z_t^{(k)} Z_{t-r}^{(k)} \right\}$$

$$\gamma_r^{(k)} = \sum_{i=0}^k \sum_{j=0}^k (-1)^{i+j} \binom{k}{i} \binom{k}{j} E(Z_{t-j} Z_{t-r-i})$$

Rearranging,

$$\gamma_r^{(k)} = \sum_{\substack{m=-k \\ i-j=m \\ 0 \leq i \leq k \\ 0 \leq j \leq k}}^k (-1)^m \binom{k}{i} \binom{k}{j} \gamma_{r+i-j}^{(0)}$$

We know that for positive integers  $a, b, n$

$$\binom{a}{0} \binom{b}{n} + \binom{a}{1} \binom{b}{n-1} + \dots + \binom{a}{n} \binom{b}{0} = \binom{a+b}{n}$$

and we observe that

$$= \sum_{\substack{i-j=m \\ 0 \leq i \leq k \\ 0 \leq j \leq k}} \binom{k}{i} \binom{k}{j} = \binom{k}{m} \binom{k}{0} + \binom{k}{m+1} + \dots + \binom{k}{k} \binom{k}{k+m}$$

$$\begin{aligned}
&= \begin{bmatrix} k \\ k-m \end{bmatrix} \begin{bmatrix} k \\ 0 \end{bmatrix} + \begin{bmatrix} k \\ k-m-1 \end{bmatrix} \begin{bmatrix} k \\ k-1 \end{bmatrix} + \cdots + \begin{bmatrix} k \\ 0 \end{bmatrix} \begin{bmatrix} k \\ k-m \end{bmatrix} \\
&= \begin{bmatrix} 2k \\ k-m \end{bmatrix}
\end{aligned}$$

Therefore,

$$\gamma_r^{(k)} = \begin{bmatrix} 2k \\ k \end{bmatrix} \gamma_r^{(0)} + \sum_{m=1}^k \begin{bmatrix} 2k \\ k-m \end{bmatrix} (-1)^m \left[ \gamma_{r+m}^{(0)} + \gamma_{r-m}^{(0)} \right]$$

and we can obtain an expression for  $\rho_1^{(k)}$  setting  $r = 1$

$$\rho_1^{(k)} = \frac{- \begin{bmatrix} 2k \\ k-1 \end{bmatrix} + \rho_1 \left( \begin{bmatrix} 2k \\ k \end{bmatrix} + \begin{bmatrix} 2k \\ k-2 \end{bmatrix} \right) - \rho_2 \left( \begin{bmatrix} 2k \\ k-1 \end{bmatrix} + \begin{bmatrix} 2k \\ k-3 \end{bmatrix} \right) + \cdots + (-1)^k \rho_{k+1}}{\begin{bmatrix} 2k \\ k \end{bmatrix} - 2\rho_1 \begin{bmatrix} 2k \\ k-1 \end{bmatrix} + 2\rho_2 \begin{bmatrix} 2k \\ k-2 \end{bmatrix} - 2\rho_3 \begin{bmatrix} 2k \\ k-3 \end{bmatrix} + \cdots + (-1)^k 2\rho_k} \quad (3-2)$$

For  $r \geq 1$ , we can obtain from the basic relationships

$$\begin{aligned}
\rho_r^{(k+1)} &= \frac{E \{ Z_t^{(k+1)} Z_{t-r}^{(k+1)} \}}{E \{ (Z_t^{(k+1)})^2 \}} \\
&= \frac{E \{ (Z_t^{(k)} - Z_{t-1}^{(k)}) (Z_{t-r}^{(k)} - Z_{t-r-1}^{(k)}) \}}{E \{ (Z_t^{(k)} - Z_{t-1}^{(k)})^2 \}} \\
&= \frac{2 \gamma_r^{(k)} - \gamma_{r+1}^{(k)} - \gamma_{r-1}^{(k)}}{2 \gamma_0^{(k)} - 2 \gamma_1^{(k)}}
\end{aligned}$$

so that

$$2 \rho_r^{(k+1)} (1 - \rho_1^{(k)}) = 2 \rho_r^{(k)} - \rho_{r+1}^{(k)} - \rho_{r-1}^{(k)}$$

and

$$\rho_{r+1}^{(k)} = - \rho_{r-1}^{(k)} + 2 \rho_r^{(k)} - 2 \rho_r^{(k+1)} (1 - \rho_1^{(k)}) \quad (3.3)$$

### 3.2 Expected Values for D-Processes

When the underlying process  $\{Z_t\}$  is Gaussian, the expected number of zero crossings,  $E D_{k,N}$ , can be evaluated directly as a function of  $\rho_1^{(k-1)}$ , the first correlation coefficient of  $\{Z_t^{(k-1)}\}$ .

We observe that

$$d_t^{(k)} = (X_t^{(k)} - X_{t-1}^{(k)})^2$$

and that

$$\begin{aligned} E d_t^{(k)} &= E \{(X_t^{(k)} - X_{t-1}^{(k)})^2\} \\ &= E \{(X_t^{(k)})^2 + (X_{t-1}^{(k)})^2 - 2 X_t^{(k)} X_{t-1}^{(k)}\} \\ &= E \{X_t^{(k)}\} + E \{X_{t-1}^{(k)}\} - 2 E \{X_t^{(k)} X_{t-1}^{(k)}\} \end{aligned} \quad (3.4)$$

since the  $x$ -processes are 0/1-valued.

The assumption that  $\{Z_t\}$  is zero-mean Gaussian implies that  $\{Z_t^{(k)}\}$  is also zero-mean Gaussian,  $k = 1, 2, \dots$ , since from (2.1) the higher difference processes are linear combinations of the 0-order process. Then

$$\begin{aligned} E \{X_t^{(k)}\} &= P \{X_t^{(k)} = 1\} \\ &= P \{Z_t^{(k-1)} \geq 0\} \\ &= 1/2 \end{aligned}$$

and

$$\begin{aligned} E \{X_t^{(k)} X_{t-1}^{(k)}\} &= P \{(Z_t^{(k-1)} \geq 0) \text{ and } (Z_{t-1}^{(k-1)} \geq 0)\} \\ &= 1/4 + (1/2\pi) \sin^{-1} \rho_1^{(k-1)} \end{aligned} \quad (3.5)$$

Thus we have

$$E d_t^{(k)} = 1/2 - (1/\pi) \sin^{-1} \rho_1^{(k-1)}$$

so

$$\begin{aligned} E D_{k,N} &= (N-1) E d_t^{(k)} \\ &= (N-1) [1/2 - (1/\pi) \sin^{-1} \rho_1^{(k-1)}] \end{aligned} \quad (3.6)$$

or

$$\rho_1^{(k-1)} = \cos \left[ \frac{\pi E D_{k,N}}{N-1} \right] \quad (3.7)$$

Values of  $E D_{k,N}$  for selected processes with  $N=1000$  are shown in Tables 4-4 A,B,C,D.

The relationship (3.7) together with (3.2) and (3.3) indicates that the expected zero-crossing rates are uniquely determined by the correlation structure of the underlying  $Z^{(0)}$ -process.

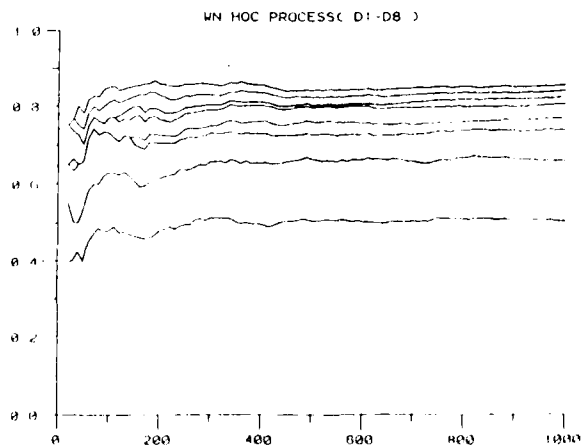
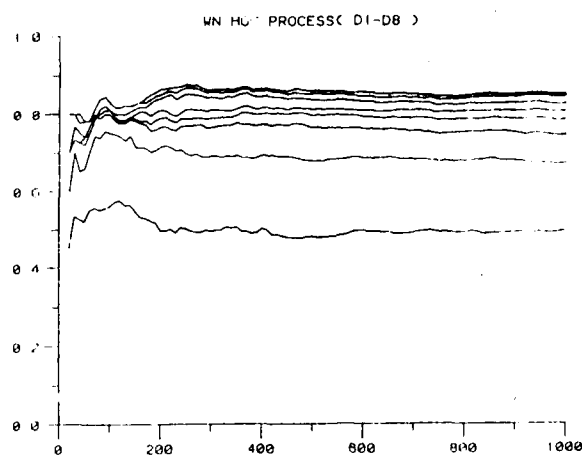
### 3.3 Convergence of HOC-processes

It is instructive to consider the graphs of some HOC-processes. Figures 3.1-3.3 show the graphs of several HOC-processes obtained from both stationary as well as nonstationary time series. The first striking observation is that the HOC-processes seem to converge monotonically to a limiting HOC-process and that the convergence is rather fast. This is the vertical convergence mentioned earlier, and one that has been observed in numerous cases as already noted in Kedem and Martin (1987). The horizontal convergence is observed in the figures at least in the stationary cases. We shall now discuss these observations more formally below.

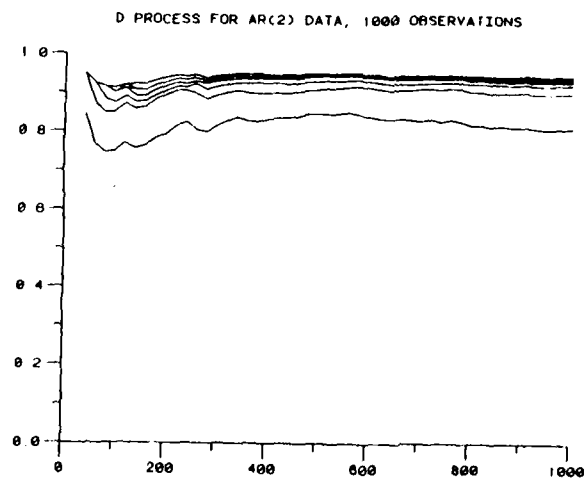
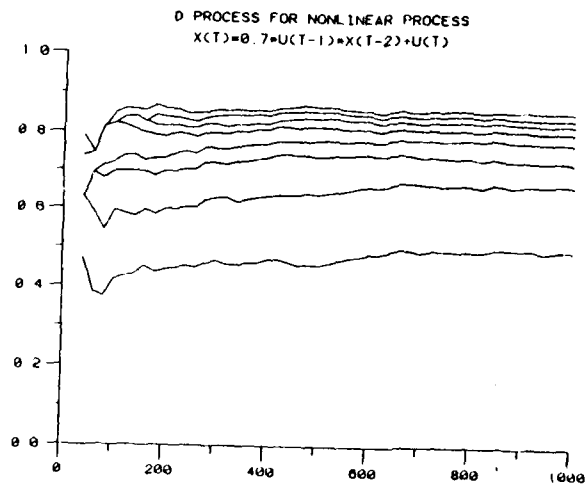
A basic fact which underlines the vertical convergence is that

$$D_{k,N} \leq D_{k+1,N} + 1. \quad (3.8)$$

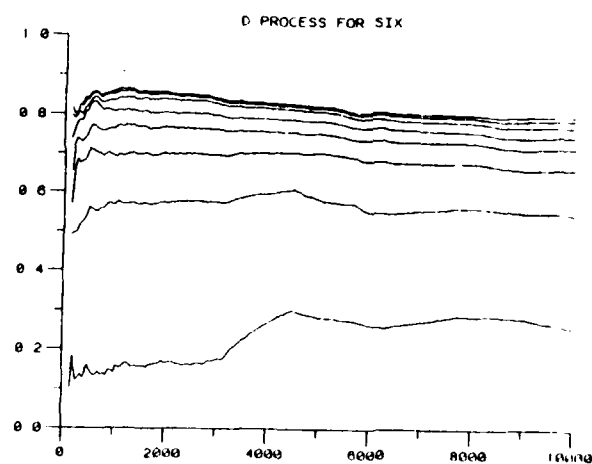
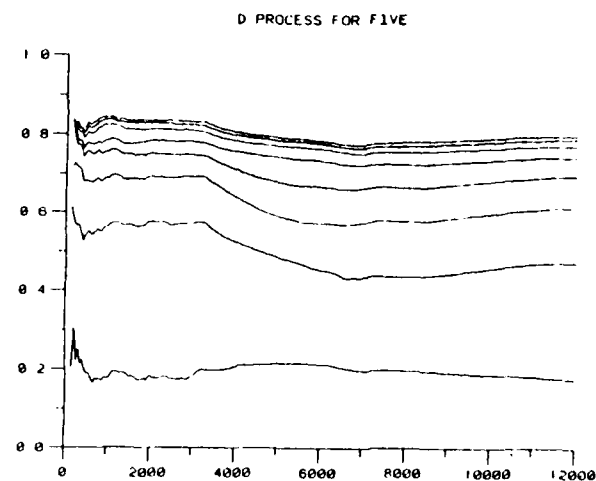
From this inequality we can show that for strictly stationary processes the expected HOC constitute a monotone sequence.



**Figure 3.1.** HOC-processes obtained from two white noise time series.



**Figure 3.2.** HOC-processes obtained from two stationary Gaussian time series.



**Figure 3.3.** HOC-processes obtained from the utterances of the English words "Five" and "six". The original time series are non-stationary.



Theorem 3.3.1. Let  $\{Z_t\}$  be a strictly stationary process. Then for every  $N$

$$E D_{k,N} \leq E D_{k+1,N}, \quad k = 1, 2, \dots$$

Proof: Define

$$\lambda_1^{(k)} = P(X_t^{(k)} = 1 | X_{t-1}^{(k)} = 1), \quad p^{(k)} = P(X_t^{(k)} = 1)$$

Then

$$E D_{k,N} = 2(N-1) p^{(k)} (1 - \lambda_1^{(k)}),$$

and together with (3.8)

$$\frac{2(N-1)p^{(k)}(1-\lambda_1^{(k)})}{N-1} \leq \frac{2(N-1)p^{(k+1)}(1-\lambda_1^{(k+1)})}{N-1} + \frac{1}{N-1}$$

or, as  $N \rightarrow \infty$

$$2p^{(k)}(1-\lambda_1^{(k)}) \leq 2p^{(k+1)}(1-\lambda_1^{(k+1)}).$$

Therefore

$$E D_{k,N} = 2(N-1)p^{(k)}(1-\lambda_1^{(k)}) \leq 2(N-1)p^{(k+1)}(1-\lambda_1^{(k+1)}) = E D_{k+1,N}$$

□

Theorem 3.3.1 obviously covers stationary Gaussian processes, but it is only a first step in the explanation of the vertical convergence. For a general result we may appeal to the theory of point processes as outlined in Kallenberg (1976).

It is convenient to restrict attention to the set  $[0, \infty)$  equipped with the Borel field  $\mathcal{B}$ . Let  $\mathcal{B}$  be the class of subsets consisting of all bounded sets in  $\mathcal{B}$ . The set of all locally finite measures that are nonnegative and integer valued is denoted by  $\mathcal{N}$ . Assume that the process  $\{Z_t\}$  is defined over the

probability space  $(\Omega, \mathcal{A}, P)$  and introduce in  $\mathcal{N}$  the  $\sigma$ -algebra  $\mathcal{F}$  generated by the mappings  $\mu \rightarrow \mu(B)$ ,  $\mu \in \mathcal{N}$ ,  $B \in \mathbb{B}$ . By a point process on  $[0, \infty)$  we mean any measurable mapping from  $(\Omega, \mathcal{A}, P)$  into  $(\mathcal{N}, \mathcal{F})$ . With an obvious extension of the previous notation, let  $D_k(B)$ ,  $B \in \mathbb{B}$ , be the number of zero-crossings generated by  $v^{k-1}Z_t$  in  $B$ . Then for each realizations of  $\{Z_t\}$ ,  $D_k(\cdot)$  defines a measure in  $\mathcal{N}$  and so, for each fixed  $k$ ,  $D_k(\cdot)$  is a point process. The vertical convergence can be explained with the help of the sequence of point processes  $\{D_k(\cdot)\}$ . We need the following combinatorial lemma.

**Lemma 3.3.1.** For any process  $\{Z_t\}$  there exists a  $t_0$ ,  $1 \leq t_0 \leq N$  and  $t_0$  depends on  $j$ ,  $j=1, 2, \dots$ , such that for every  $N$

$$\sum_{\substack{t=1 \\ t \neq t_0}}^N d_t^{(j)} \leq \sum_{\substack{t=1 \\ t \neq t_0}}^N d_t^{(j+1)} \quad (3.9)$$

with probability one.

**Proof:** Fix  $j$ . Suppose  $d_1^{(j)} + \dots + d_N^{(j)} = 0$ ; then we are done.

So suppose  $d_1^{(j)} + \dots + d_N^{(j)} > 0$ . Then there exists at least one symbol change in  $\{X_t^{(j)}\}$ ,  $1 \leq t \leq N$ . Note that in general we have the implication

$$\{X_{t-1}^{(j)} \neq X_t^{(j)}\} \subset \{X_t^{(j+1)} \neq X_t^{(j)}\}.$$

It follows that there exists  $t_1$  such that

$$X_{t_1}^{(j+1)} = X_{t_1}^{(j)}.$$

Therefore each symbol change in  $\{X_t^{(j)}\}$  produces at least one symbol change in  $\{X_t^{(j+1)}\}$  for  $t_1 < t \leq N$ . On the other hand by

going from  $j$  to  $j+1$  we may lose, by (3.8), at most one zero-crossing for  $1 \leq t \leq t_1$ . It follows that there exists a  $t_0$  which depends on  $j$  such that (3.9) holds.  $\square$

The lemma says that in the binary sequence  $\{X_t^{(j+1)}\}$ ,  $1 \leq t \leq N$ , there are  $N-1$  locations that give at least as many symbol changes as the corresponding locations in  $\{X_t^{(j)}\}$ ,  $1 \leq t \leq N$ , while no information is available about the remaining location which we denote by  $t_0$ . With the help of the lemma we can now prove the weak convergence of the sequence of point processes  $\{D_k(\cdot)\}$ .

Theorem 3.3.2. Let  $\{Z_t\}$  be a strictly stationary process and let  $D(\cdot)$  be the point process corresponding to the limiting HOC-process. Then  $D_k(\cdot) \rightarrow D(\cdot)$  with respect to the vague topology on  $\mathcal{M}$ .

Proof: For any  $B \in \mathcal{B}$  we can have at most a finite number of integer points and thus at most a finite number of zero-crossings by  $\{v^k Z_t\}$  for any  $k$ . Therefore

$$\lim_{r \rightarrow \infty} \limsup_{k \rightarrow \infty} P(D_k(B) > r) = 0$$

Next, by Theorem 3.3.1, for any  $I = (a, b]$ ,  $a, b$  finite, we have by monotonicity

$$E D_k(I) \rightarrow E D(I), \quad k \rightarrow \infty.$$

Third, consider the higher order crossings  $D_{k,N}$  and  $D_{k+1,N}$ . Then by the lemma there exist  $t_0$ ,  $D'_k$  and  $D'_{k+1}$  such that

$$\begin{aligned} D_{k,N} &= D'_k + d_{t_0}^{(k)} \\ D_{k+1,N} &= D'_{k+1} + d_{t_0}^{(k+1)} \end{aligned}$$

and

$$D'_k \leq D'_{k+1} \quad \text{surely.}$$

By strict stationarity and since  $d_t^{(k)}$  is binary

$$P\left[d_{t_0}^{(k)} \geq 1\right] \leq P\left[d_{t_0}^{(k+1)} \geq 1\right]$$

Therefore

$$P\left[D'_k + d_{t_0}^{(k)} \geq 1\right] \leq P\left[D'_{k+1} + d_{t_0}^{(k+1)} \geq 1\right]$$

or

$$P\left[D_{k,N} = 0\right] \geq P\left[D_{k+1,N} = 0\right]$$

and so the sequence  $P(D_{k,N} = 0)$  converges monotonically as  $k \rightarrow \infty$  for any  $N$ . By appealing to Theorem 4.7 in Kallenberg (1976) it follows that  $D_k(\cdot)$  converges in distribution to  $D(\cdot)$ .  $\square$

For the horizontal convergence we require ergodicity.

**Theorem 3.3.3.** Let  $\{Z_t\}$  be an ergodic zero-mean stationary Gaussian process. Then for every  $k, k=1,2,\dots$ ,

$$\lim_{N \rightarrow \infty} \frac{D_{k,N}}{N} = \frac{1}{\pi} \cos^{-1}(\rho_1^{(k-1)}) \quad \text{a.s.}$$

Proof: Apply (3.7).  $\square$

This explains the convergence to straight horizontal lines observed in the figures. It follows that for Gaussian white noise the straight lines are at levels  $\frac{1}{\pi} \cos^{-1}(-k/(k+1))$ ,  $k=0,1,2,\dots$ .

When both the order and the size increase we have convergence towards the highest possible frequency in the spectrum if the latter exists. More precisely we have. Define

$$\bar{D}_j(N) \equiv \frac{D_{j,N}}{N}.$$

Also, the normalized spectral measure of  $\{\nabla^j Z_t\}$  is given by

$$\nu_j(d\omega) = \frac{\left[\sin \frac{1}{2} \omega\right]^{2j} dF(\omega)}{\int_{-\pi}^{\pi} \left[\sin \frac{1}{2} \lambda\right]^{2j} dF(\lambda)}$$

where  $F$  is the spectral distribution function of  $\{Z_t\}$ . Let  $\delta_u$  be the point mass at  $u$ .

**Theorem 3.3.4.** Let  $\{Z_t\}_{t=1}^{\infty}$  be a zero mean stationary Gaussian process, and suppose  $\omega^* \leq \pi$  is the highest frequency in the spectral support. If  $\lim_{N \rightarrow \infty} \bar{D}_j(N) \leq \omega^*/\pi$  with probability one,  $j = 1, 2, \dots$ , then

$$\lim_{j \rightarrow \infty} \lim_{N \rightarrow \infty} \bar{D}_j(N) = \frac{\omega^*}{\pi} \text{ a.s.}$$

**Proof.** From Kedem and Slud (1982)  $\nu_j \Rightarrow \frac{1}{2} \delta_{-\omega^*} + \frac{1}{2} \delta_{\omega^*}$ ,  $j \rightarrow \infty$ . But the Gaussian assumption implies that

$$\cos(\pi E \bar{D}_j(N)) = \int_{-\pi}^{\pi} \cos(\omega) \nu_j(d\omega) \rightarrow \cos(\omega^*), \quad j \rightarrow \infty.$$

Therefore  $E \bar{D}_j(N) \rightarrow \omega^*/\pi$ ,  $j \rightarrow \infty$ , uniformly in  $N$ .

By strict stationarity we can define the a.s. limit

$$c_j = \lim_{N \rightarrow \infty} \bar{D}_j(N).$$

Then by Lemma 1  $c_j \leq c_{j+1}$  a.s., and so  $\lim_{j \rightarrow \infty} c_j = c \leq \omega^*/\pi$  a.s.

Thus by bounded convergence  $E(c_j) \rightarrow E(c)$ ,  $j \rightarrow \infty$ . Strict stationarity and Fatou's Lemma yield

$$E(c_j) \geq \limsup_{N \rightarrow \infty} E \bar{D}_j(N).$$

Since  $E c_j \rightarrow E c$  and  $E \bar{D}_j(N) \rightarrow \omega^*/\pi$ ,

$$E(c) \geq \omega^* / \pi.$$

But  $c \leq \omega^* / \pi$ . Therefore  $c = \omega^* / \pi$  a.s. □

Corollary 3.3.1. If  $\omega^* = \pi$  then

$$\lim_{j \rightarrow \infty} \lim_{N \rightarrow \infty} \bar{D}_j(N) = 1.$$

We close this section by noting that Theorem 3.3.2 which addresses the vertical convergence is somewhat more general than the Higher Order Crossings Theorem proved in Kedem and Slud (1982) and which says that for a strictly stationary process with spectral distribution function  $F$  supported at  $\pi$ , as  $k \rightarrow \infty$ ,

$$\left\{ X_t^{(k)} \right\} \rightarrow \begin{array}{ll} \dots 010101 \dots & \text{w.p. } 1/2 \\ \dots 101010 \dots & \text{w.p. } 1/2 \end{array}.$$

Theorem 3.3.2 does not presuppose that  $\{Z_t\}$  possesses moments at all, and strictly speaking, it is not a spectral result but can be viewed as such in a generalized sense. It is important to observe that a process need not possess moments of any order and yet its HOC-processes possess moments of all orders.

## 4.0 ESTIMATION OF VARIANCE OF D-PROCESSES

### 4.1 Direct Computation

In order to use zero-crossing statistics for testing, one needs at least variance estimates for the  $D_{k,N}$ . These estimates are in general functions of fourth order orthant probabilities in  $Z_t$  which are not easily obtainable.

In addition calculation of the fourth order orthants leads to a difficult computational problem since the error appears to increase rapidly with  $N$ .

In what follows we discuss in some detail the problem of approximating the variance of  $D_{k,N}$ . Fortunately, it is possible to obtain a very reasonable approximation under a certain condition.

## 4.2 Simulation

As an alternative to direct computation of the variance, approximations to the variance of  $D_{k,N}$  can be obtained from computer simulations. The computer is programmed to generate realizations of a specified process from which counts of the higher order zero-crossings are made. Counts are obtained for a number of realizations, and the sample mean and variance of these statistics are calculated.

In our study, we have used computer simulation results as a comparison for variance estimates obtained by analytical methods. We have implemented these simulations both on a large mainframe computer (UNIVAC 1108) using FORTRAN, and on an IBM personal computer using TurboPASCAL. We note that while the execution time is considerably faster on the mainframe, running time on a PC is not prohibitively slow (approximately 20 minutes for 100 iterations with 1000 observations and six levels of higher-order crossings) and offers some advantages of flexibility and lower cost.

Some examples of simulation results are shown in Tables 4-1A,B,C and 4-2A,B,C. The results agree with an earlier simulation reported in Kedem (1987)



TABLE 4-1A

Simulation Results for  $D_{1,N}$   
 White Noise Process  
 (100 iterations)

N	$\hat{E} D_{1,N}$	$\left[ \hat{\text{Var}}(D_{1,N}) \right]^{1/2}$
10	4.71	1.54
20	9.96	2.03
50	24.93	3.46
100	49.61	4.92
200	99.46	7.84
500	249.76	10.16
1000	499.15	15.28

TABLE 4-1B

Simulation Results for  $D_{1,N}$   
 AR(1) Process  $\phi=0.5$   
 (100 iterations)

N	$\hat{E} D_{1,N}$	$\left[ \hat{\text{Var}}(D_{1,N}) \right]^{1/2}$
10	3.65	1.44
20	6.88	2.55
50	16.52	3.53
100	32.77	4.89
200	67.43	6.88
500	167.56	11.64
1000	336.89	15.91

TABLE 4-1C

Simulation Results for  $D_{1,N}$ AR(1) Process  $\phi=0.2$   
(100 iterations)

N	$\hat{E} D_{1,N}$	$\left[\hat{\text{Var}}\left[D_{1,N}\right]\right]^{1/2}$
10	4.40	1.63
20	8.58	2.10
50	22.56	3.60
100	43.51	5.08
200	86.78	7.05
500	217.84	10.11
1000	437.83	15.76

TABLE 4-2A

Simulation Results for  $D_{k,N}$   
 White Noise Process  
 (N=1000, 100 iterations)

k	$\hat{E} D_{k,N}$	$\left[\hat{\text{Var}}\left(D_{k,N}\right)\right]^{1/2}$
1	500.16	16.23
2	668.00	12.05
3	732.26	11.86
4	768.49	11.24
5	793.20	10.93
6	811.74	10.64

TABLE 4-2B

Simulation Results for  $D_{k,N}$   
 AR(1) Process  $\phi=0.5$   
 (N=1000, 100 iterations)

k	$\hat{E} D_{k,N}$	$\left[\hat{\text{Var}}\left(D_{k,N}\right)\right]^{1/2}$
1	335.16	16.58
2	578.87	13.54
3	683.52	13.94
4	739.10	12.00
5	773.83	11.64
6	797.95	10.64

TABLE 4-2C

Simulation Results for  $D_{k,N}$   
AR(1) Process  $\phi=0.2$   
(N=1000, 100 iterations)

k	$\bar{E} D_{k,N}$	$\left[ \bar{\text{Var}} \left( D_{k,N} \right) \right]^{1/2}$
1	438.37	17.93
2	629.43	14.82
3	708.89	12.58
4	754.20	12.13
5	783.28	11.02
6	804.30	10.39

### 4.3 Approximation Assuming Negligible Cumulants

For any stationary  $\{Z_t\}$  (not necessarily Gaussian) with known mean  $\mu$ , define

$$C_h = C_{-h} = \frac{1}{T-h} \sum_{t=1}^{T-h} (Z_t - \mu)(Z_{t+h} - \mu), \quad (4.1)$$

$$h = 0, 1, \dots, T-1$$

which is an unbiased estimate of  $\gamma_h$ .

It is shown in Anderson (1971, p.452) that

$$(T-h) \text{ Var } C_h = \sum_{r=-(T-h-1)}^{T-h-1} \left[ 1 - \frac{|r|}{T-h} \right] [\gamma_r^2 + \gamma_{r+h} \gamma_{r-h} + \kappa(h, -r, h-r)] \quad (4.2)$$

where  $\kappa$  is the cumulant function

$$\begin{aligned} \kappa(h, r, s) &= E \{ (Z_t - \mu)(Z_{t+h} - \mu)(Z_{t+r} - \mu)(Z_{t+s} - \mu) \} \\ &= [\gamma_h \gamma_{r-s} + \gamma_r \gamma_{h-s} + \gamma_s \gamma_{h-r}] \\ t &= \dots, 1, 0, 1, \dots \end{aligned}$$

When  $\{Z_t\}$  is Gaussian  $\kappa$  vanishes.

We observe that

$$\begin{aligned} D_{1,N} &= \sum_{i=2}^N d_i^{(1)} \\ &= \sum_{i=2}^N \left[ x_i^{(1)} - x_{i-1}^{(1)} \right]^2 \\ &= \sum_{i=2}^N \left[ x_i^{(1)} \right]^2 + \sum_{i=2}^N \left[ x_{i-1}^{(1)} \right]^2 - 2 \sum_{i=2}^N \left[ x_i^{(1)} x_{i-1}^{(1)} \right] \\ &= \sum_{i=2}^N \left[ x_i^{(1)} - 1/2 \right] \left[ x_{i-1}^{(1)} - 1/2 \right] + (N-1)/2 \end{aligned}$$

$$= - (N-1) 2C_1^{X^{(1)}} + \frac{N-1}{2}$$

where  $C_h^{X^{(1)}}$  is the function (4.1) for the  $X^{(1)}$ -process. Thus,

$$\text{Var} \left[ \frac{D_{1,N}}{N-1} \right] = 4 \text{Var} \left[ C_1^{X^{(1)}} \right]. \quad (4.3)$$

If we can assume that the cumulants are negligible for the processes we are interested in, then (4.2) is easy to evaluate and can be used to give estimates of the variance of  $D_{k,N}$  by using (4.3).

We calculated estimates of the variance of  $D_{1,N}$  for Gaussian White noise and for Gaussian AR(1) processes with parameters 0.5 and 0.2. The results are shown in Table 4-3. For white Gaussian noise the cumulants are 0, so (4.2) works exactly for this process. However, the estimates for the AR(1) processes appear to overestimate the variance compared to the simulation results in Table 4-2B,C, and the differences are larger as the AR(1) process moves away from white noise. Thus we conclude that this method does not provide good variance estimates for processes other than white noise. In other words, the cumulant function contribution to (4.2) is appreciable.

TABLE 4-3

Results of Approximation Based on the modified  
Expression (4.2) For  $(\text{Var}(D_{1,N}))^{1/2}$

N	Process		
	White Noise	AR(1) $\phi = 0.5$	AR(I) $\phi = 0.2$
10	1.50	1.95	1.57
20	2.18	2.88	2.28
50	3.50	4.66	3.67
100	4.97	6.64	5.22
200	7.05	9.44	7.41
500	11.17	14.95	11.73
1000	15.80	21.16	16.60

#### 4.4 Approximation for White Noise

Kedem and Reed (1986) showed that for an  $m$ -dependent stationary Gaussian process  $\{Z_t\}$  with  $m$  in the spectral support and for fixed  $N$

$$\lim_{k \rightarrow \infty} \frac{\text{var}(D_{k,N})}{(N-1) \lambda_1^{(k)} (1 - \lambda_1^{(k)})} = 1 \quad (4.4)$$

where

$$\begin{aligned} \lambda_1^{(k)} &= P\{x_t^{(k)} = 1 \mid x_{t-1}^{(k)} = 1\} \\ &= 1/2 + 1/n \sin^{-1} p_1^{(k-1)} \quad k=1, 2, \dots \end{aligned}$$

Thus approximation to the variance of  $D_{k,N}$  is provided by

$$\text{Var}(D_{k,N}) = (N-1) \lambda_1^{(k)} (1 - \lambda_1^{(k)}) \quad (4.5)$$

This method is incorporated in the method using first order Markov approximation (Section 4.5). We may view (4.5) as 0'th order approximation.

#### 4.5 Approximation assuming first order Markov process.

The difficulties in the computation of  $\text{Var}(D_{k,N})$  can to a large extent be passed by assuming that the binary process  $\{d_t^{(k)}\}$ ,  $t=0, 1, \dots$ , is a Markov chain for each fixed  $k$ . This idea has been suggested and implemented in Kedem (1987) where it is shown that the resulting approximation is surprisingly good. The method is summarized in the following algorithm.

Given  $\rho_1, \dots, \rho_k, \rho_{k+1}$ , we can compute  $E D_{k,N}$  exactly and approximate  $\text{Var}(D_{k,N})$  by following steps 1-7.

Step 1. Obtain  $\rho_1^{(k)}$  from (3.2).



Step 2. Obtain  $\rho_2^{(k)}$  from (3.3).

Step 3. Compute

$$\lambda_j^{(k)} = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \rho_j^{(k-1)}, \quad j = 1, 2$$

Step 4. Compute

$$p^{(k)} = 1 - \lambda_1^{(k)}, \quad q^{(k)} = 1 - p^{(k)}$$

Step 5. Compute

$$\nu^{(k)} = \frac{1 - 2\lambda_1^{(k)} + \lambda_2^{(k)}}{2(1 - \lambda_1^{(k)})}$$

Step 6. The exact expected value of  $D_{k,N}$  is given by

$$E D_{k,N} = (N-1)p^{(k)}$$

Step 7. The variance of  $D_{k,N}$  is approximated by

$$\text{Var}(D_{k,N}) = (N-1)p^{(k)}q^{(k)} + \frac{2p^{(k)}q^{(k)}(\nu^{(k)} - p^{(k)})}{(1 - \nu^{(k)})} \left\{ (N-1) - V_{k,N} \right\} \quad (4.6)$$

where

$$V_{k,N} = \frac{q^{(k)} [1 - ((\nu^{(k)} - p^{(k)})/q^{(k)})^{N-1}]}{(1 - \nu^{(k)})}$$

The variance expression (4.6) can be recognized as the variance of

$$d_2^{(k)} + \dots + d_N^{(k)}$$

under the assumption that  $\{d_t^{(k)}\}$  is a two state stationary Markov chain with parameters  $p^{(k)}$  and  $\nu^{(k)}$ . Observe that (4.6) is (4.5) plus a "correction" term. Tables 4.4A - 4.4D show some computations using formula (4.6). These results agree closely with simulation results. Subroutine DVAR attached at the end, uses the above algorithm in the computation of (4.6).

TABLE 4-4A

Results of First Order Markov  
Approximation for White Noise  
(N = 1000)

k	E $D_{k,N}$	$\left[ \text{Var} \left( D_{k,N} \right) \right]^{1/2}$
1	499.50	15.80
2	666.00	13.14
3	731.55	12.17
4	769.18	11.58
5	794.37	11.16
6	812.76	10.82

TABLE 4-4B

Results of First Order Markov  
Approximation for AR(1)  $\phi = 0.5$   
(N = 1000)

k	E $D_{k,N}$	$\left[ \text{Var} \left( D_{k,N} \right) \right]^{1/2}$
1	333.00	15.76
2	579.85	13.99
3	684.69	12.74
4	729.67	12.00
5	773.82	11.49
6	797.44	11.10

TABLE 4-4C

Results of First Order Markov  
Approximation for AR(1)  $\phi = 0.2$   
(N = 1000)

k	E $D_{k,N}$	$\left[ \text{Var} \left( D_{k,N} \right) \right]^{1/2}$
1	435.47	15.82
2	630.36	13.41
3	709.85	12.39
4	754.45	11.77
5	783.61	11.33
6	804.48	10.97

TABLE 4-4D

Results of First Order Markov  
Approximation for AR(1)  $\phi = -0.8$   
(N = 1000)

k	E $D_{k,N}$	$\left[ \text{Var} \left( D_{k,N} \right) \right]^{1/2}$
1	794.37	14.74
2	855.58	11.16
3	876.17	10.10
4	887.93	9.53
5	895.97	9.16
6	901.98	8.89

## 5. Testing for white noise: an application of HOC processes

Finally, after discussing the dynamic behavior of HOC-processes in regard to order and time, we are ready to apply this interesting dynamics in tests appropriate for time series models. We concentrate on testing for white noise but the technique can be readily extended to more general tests regarding autoregressive-moving average processes.

As noted earlier, HOC-processes summarize the oscillatory characteristics of a process. Moreover, it is apparent from Figures, 3.1-3.2 that for stationary time series, HOC-processes produce sample paths which settle about straight lines even for moderate sample sizes, and that the initial rate of the vertical convergence is rather fast which is a general property of higher order crossings. Thus, in this sense, it is possible to characterize a process by a few HOC-processes observed between, say,  $N = 500$  to  $N = 1000$ . Experience shows, and this will be illustrated again below, that as few as six HOC-processes need to be considered. Based on these observations, the proposed test for white noise is to observe whether the first six HOC-processes produce sample paths which indeed fall within prescribed probability bounds derived under the hypothesis of white noise. The hypothesis of white noise is rejected as soon as at least one of the six paths cross its probability bounds. Such a scheme is akin to control processes and can be used in real time monitoring of man made systems.

Observe that under the Markov assumption,  $D_{k,N}$  has an asymptotic normal distribution so that it lies in

$$E D_{k,N} \pm 1.96 \sqrt{\text{Var}(D_{k,N})} \quad (5.1)$$

with 95% approximately, for each fixed  $k$  and large  $N$ . However as  $N$  varies we must obtain a simultaneous bound for the HOC path  $\{D_{k,N}/N\}$ ,  $N_1 \leq N \leq N_2$ . In this work we take  $N_1 = 500$ ,  $N_2 = 1000$ ,  $1 \leq k \leq 6$ . The bounds are determined from

$$E D_{k,N} \leq c \sqrt{\text{Var}(D_{k,N})}, \quad 500 \leq N \leq 1000 \quad (5.2)$$

for each  $k$ ,  $1 \leq k \leq 6$ , where  $\{\text{Var}(D_{k,N})\}^{1/2}$  is given by (4.6) assuming white noise. The constant  $c$  controls the overall power and the  $\alpha$ -level of the test. When  $c=1.96$  the test is very powerful against many alternatives but also has a high  $\alpha$ -level. In order to lower the level we widen the bounds by choosing larger values for  $c$ , a procedure that leads to more conservative bounds.

### 5.1 Power considerations and examples

Simulations were first run to estimate the power of the test for white noise as a function of  $c$ , the test width coefficient in (5.2). Three processes were simulated: white noise and two AR(1) processes with parameters 0.05 and 0.1. The resulting HOC-process paths were monitored using the white noise limits obtained from (4.6) and (5.2). Figures 5.1-5.3 show the percentage of cases failing the test for the three simulations.

From Figure 5.1 it appears that a  $c$ -value close to 3 corresponds to a level around 5%. In fact, we obtained an  $\alpha$ -level of 0.06 for  $c=2.8$ . The power corresponding to  $c=2.8$  is about 20% and 40% for  $\theta=0.05$  and  $\theta=0.1$ , respectively. This is shown in Figures 5.2, 5.3. The curves shown in the three figures were obtained by a least-squares fit.

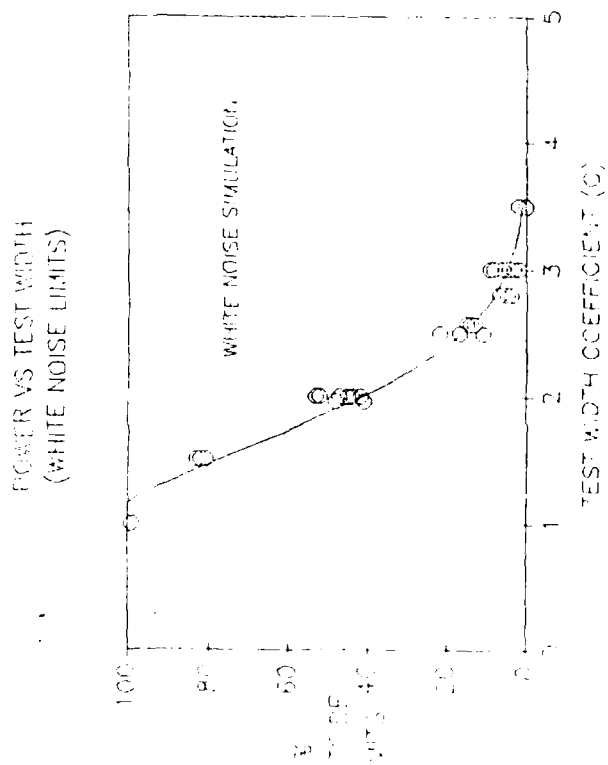


Figure 5.1. The level of the white noise test as a function of C.

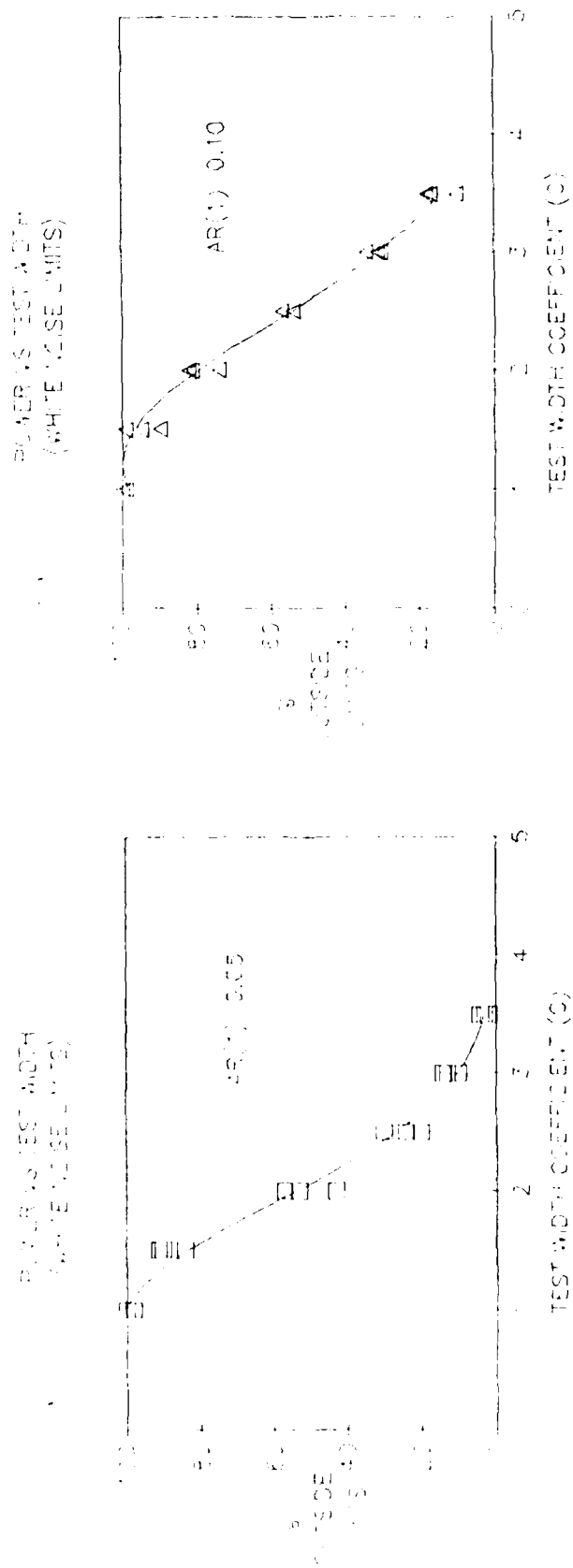


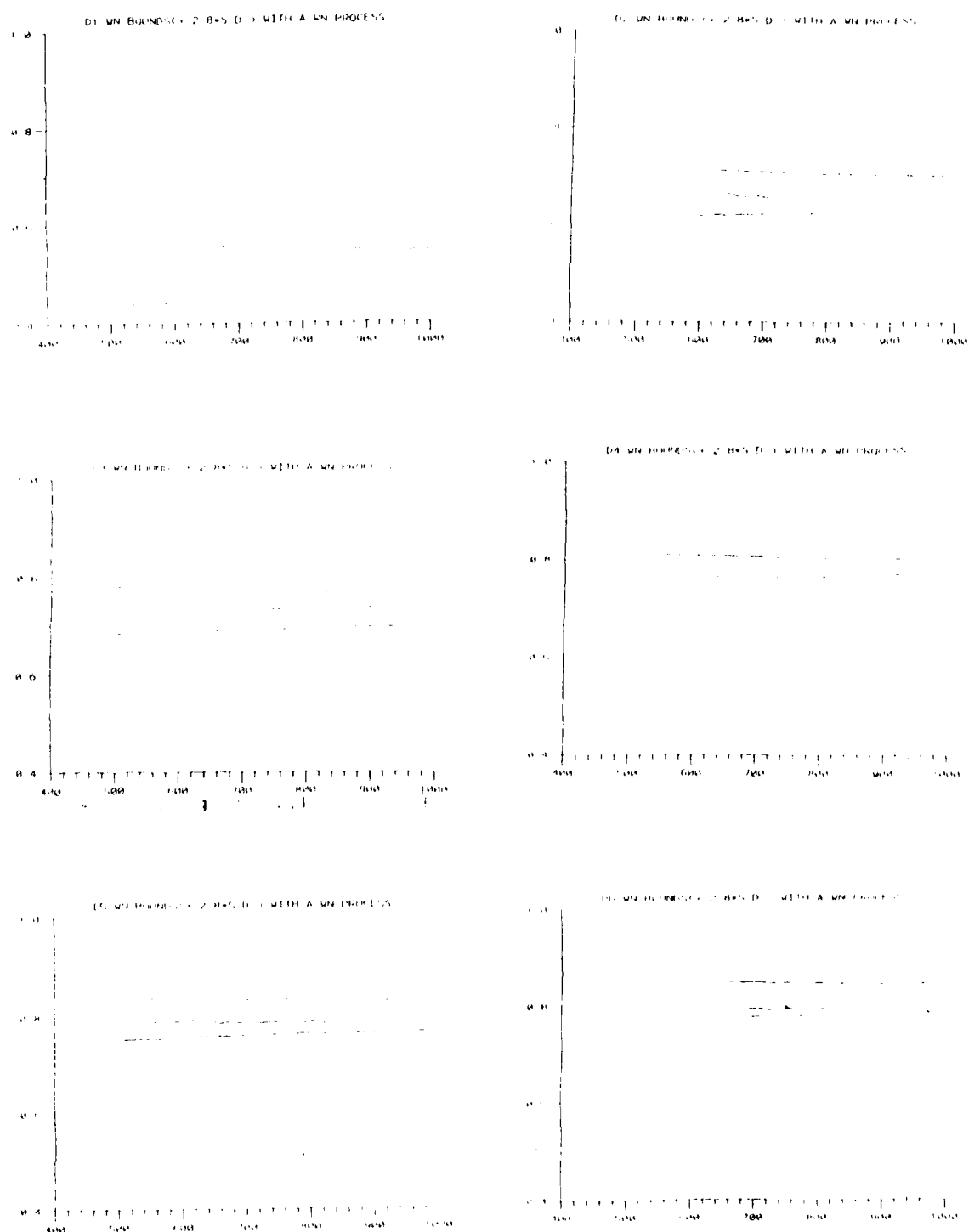
Figure 5.2 Power of the test as a function of C at an  $\alpha = 0.05$ , an  $\text{AR}(1)$ ,  $\rho = 0.1$

Table 5.1 gives the results of some power calculations obtained from 100 simulations for each alternative from ARMA(1,1). The value of  $c$  is 2.8. The further the alternative is from white noise the higher is the power as expected.

Examples which illustrate the use of the test procedure are shown in Figures 5.4-5.6. For non-white noise in many cases higher order HOC-processes resemble those from white noise and the distinction is made by low order HOC-processes. Occasionally it is the D6-path that captures the difference while the lower order cases do not.

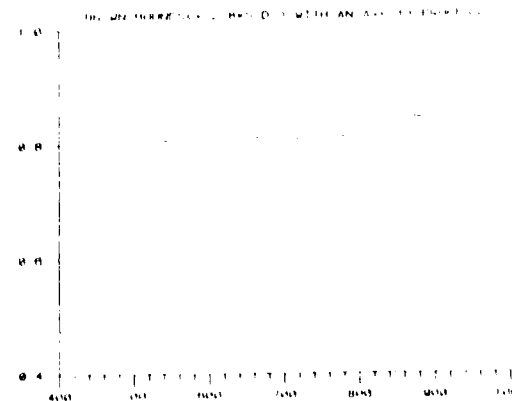
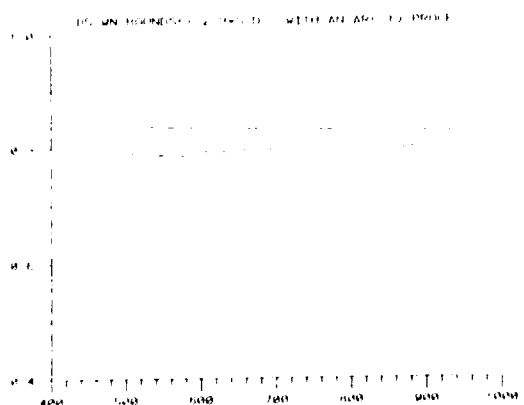
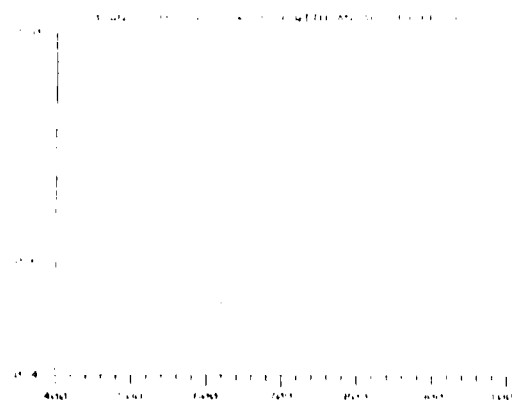
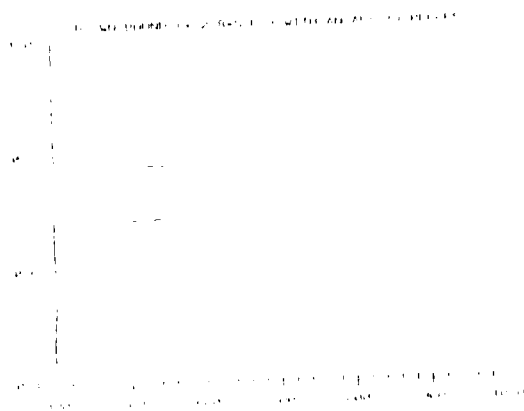
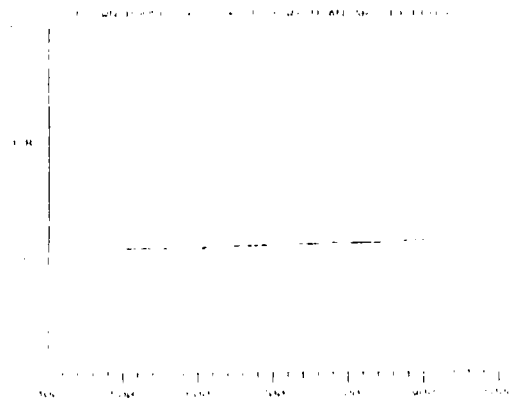
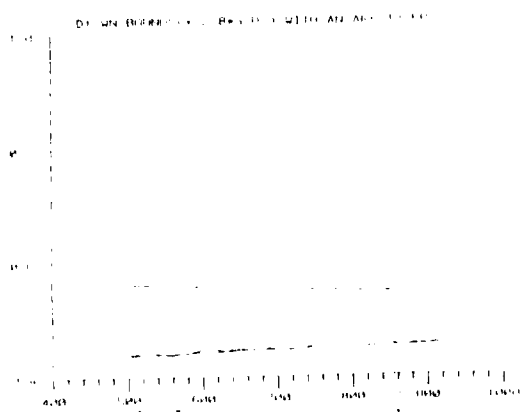
$\phi$	$\theta$	Power
0.00	0.00	0.06
0.05	0.00	0.19
0.00	0.10	0.38
0.10	0.00	0.39
0.05	0.05	0.46
0.15	0.00	0.73
0.20	0.00	0.95
0.10	0.10	0.97
0.00	0.20	1.00
0.20	0.20	1.00
0.05	0.30	1.00

Table 5.1. Power for the white noise test against alternatives from  $Z_t = \phi Z_t + u_t - \theta u_{t-1}$  where  $u_t$  are independent  $N(0,1)$  random variables.

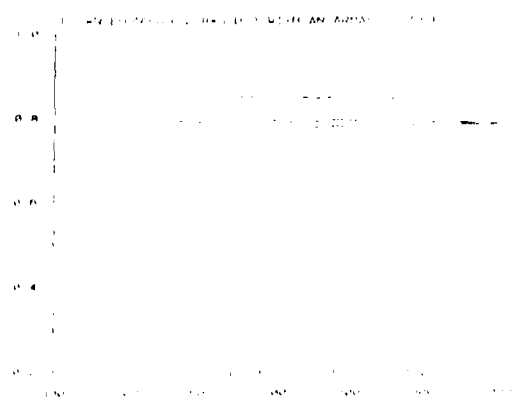
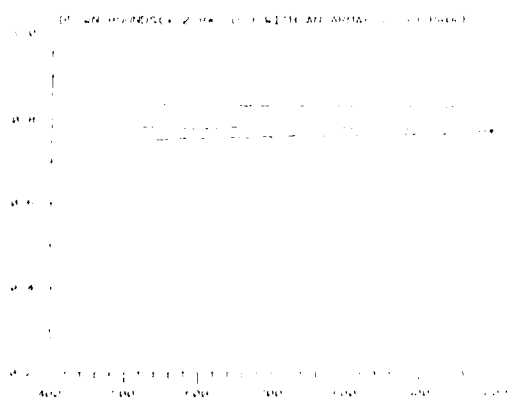
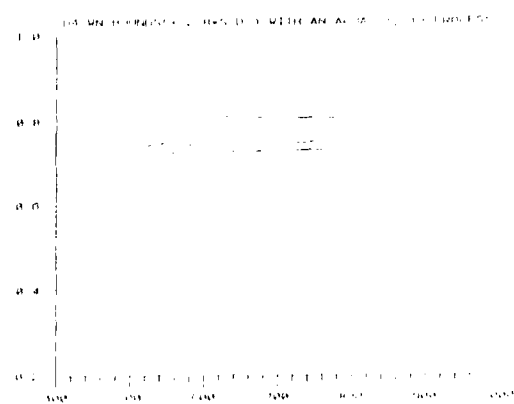
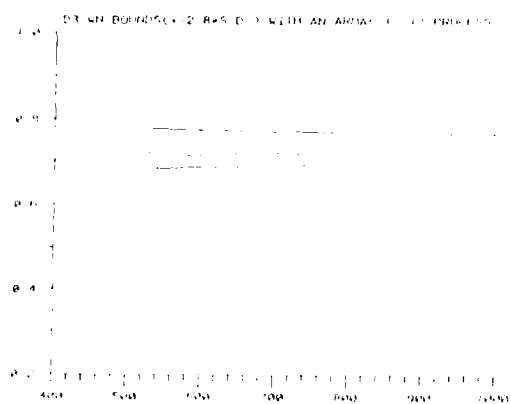
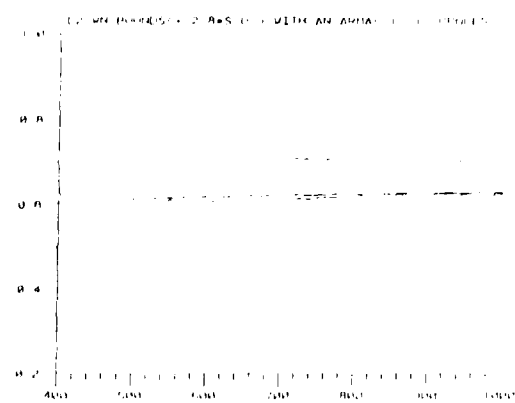
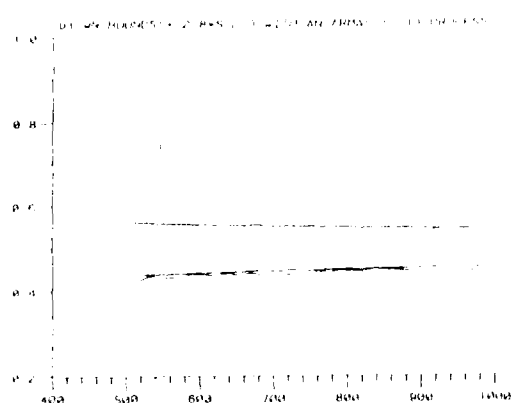


**Figure 5.4** HOC-processes from white noise. The hypothesis of white noise is accepted. Each HOC-path is within the white noise limits.





**Figure 5.5** Hoc-processes from AR(1),  $\phi=0.1$ , and white noise limits. The hypothesis of white noise is rejected.



**Figure 5.6** Hoc-processes from ARMA(1,1),  $\phi=0.1$ , and white noise limits. The hypothesis of white noise is rejected.

## 5.2 Identification of general models.

There is no reason why the above procedure cannot be extended to test for non-white models, because once a model is completely specified and its correlation structure is known as a result, (4.6) and (5.2) can be evaluated and the limits can be obtained as before. Our experimental results indicate that to maintain a reasonable test level the value of 2.8 for  $c$  in (5.2) is still a good choice. Table 5.2 gives some power calculation for testing the hypothesis that a process is AR(1) with parameter  $\phi = 0.1$ , where the alternatives are generated by ARMA(1,1) with parameters  $\phi, \theta$  as above.

$\phi$	$\theta$	Power
0.10	0.00	0.07
0.05	0.05	0.08
0.00	0.10	0.10
0.05	0.00	0.12
0.00	0.00	0.42
0.20	0.00	0.43
0.00	0.20	0.50
0.10	0.10	0.51
0.30	0.00	0.97
0.05	0.50	1.00

Table 5.2. Power of the test for testing the hypothesis that the process is AR(1) with  $\phi = 0.1$ . The alternatives are generated by different ARMA(1,1) processes.

## 6. Summary

We have outlined a test procedure for time series models based on HOC-processes. The test can be used in online situations, for example when monitoring the oscillation of signatures obtained from man made systems such as engines. As was indicated, if higher  $\alpha$ -levels can be tolerated, the power against different alternatives can increase appreciably. This can be achieved by lowering  $c$  and thus rendering the limits less conservative.

The ideas presented in this paper can be extended to cover nonstationary time series as well since the vertical convergence is quite a general phenomenon and seems to hold across a wide range of processes.

The present paper is a generalization, an application of HOC to one based on HOC-processes. For each fixed  $N$  the sequence  $\{D_{k,N}\}$ ,  $k = 1, 2, \dots$ , is the sequence of higher order crossings. It is known that the rate of increase in HOC, and hence also in HOC-processes, is a useful feature in discriminating between processes as is exemplified in Figures 3.1-3.3. Tests based on  $\{D_{1,N}, \dots, D_{6,N}\}$  for fixed  $N$  are discussed in Kedem (1987). It seems that no real advantage is gained by increasing the order  $k$  in the present use of HOC-processes amidst stationary time series. On the other hand by increasing  $N$  we can obtain a more powerful test. This pertains particularly to borderline cases that need "more time" to get out of the probability bounds as they become narrower and narrower.

Finally, the method described herein impinges on sequential analysis. This connection will be pursued elsewhere.

```

C SUBROUTINE DVAR COMPUTES E(D(K)),VAR(D(K)),K=1,...,6
C GIVEN THE FIRST 7 AUTOCORRELATIONS OF A ZERO MEAN
C STATIONARY GAUSSIAN TIME SERIES.
C R:INPUT.VECTOR CONTAINING FIRST 7 AUTOCORRELATIONS.
C N:INPUT.NET LENGTH.LENGTH OF TIME SERIES MINUS 5 (DIFFERENCES)
C RD:OUTPUT.VECTOR CONTAINING RD(K),K=0,...,6.
C   RD(K) IS 1'ST AUTOCOR. IN THE K'TH DIFFERENCE.
C   RD(K) IS COMPUTED FROM R(1),...,R(K+1)
C ED:OUTPUT.VECTOR CONTAINING EXPECTED HOC K=1,...,6
C VD:OUTPUT.VECTOR CONTAINING VARIANCES OF HOC K=1,...,6.
*
SUBROUTINE DVAR(R,N,RD,ED,VD)
REAL R(7),ED(6),VD(6),RD(0:6)
RD(0)=R(1)
RD(1)=(-1.+2.*R(1)-R(2))/(2.*(1.-R(1)))
RD(2)=(-4.+7.*R(1)-4.*R(2)+R(3))/(6.-8.*R(1)+2.*R(2))
RD(3)=(-15.+26.*R(1)-16.*R(2)+6.*R(3)-R(4))/
1 (20.-30.*R(1)+12.*R(2)-2.*R(3))
RD(4)=(-56.+98.*R(1)-64.*R(2)+29.*R(3)-8.*R(4)+R(5))/
1 (70.-112.*R(1)+56.*R(2)-16.*R(3)+2.*R(4))
RD(5)=(-210.+372.*R(1)-255.*R(2)+130.*R(3)-46.*R(4)+10.*R(5)
1 -R(6))/(252.-420.*R(1)+240.*R(2)-90.*R(3)+20.*R(4)-2.*R(5))
RD(6)=(-792.+1419.*R(1)-1012.*R(2)+561.*R(3)-232.*R(4)+67.*R(5)
1 -12.*R(6)+R(7))/(924.-1584.*R(1)+990.*R(2)-440.*R(3)+
1 132.*R(4)-24.*R(5)+2.*R(6))
DO 1 I=0,5
P1=RD(I)
R2=-1.+2.*RD(I)-2.*RD(I+1)*(1.-RD(I))
ALAM1=0.5+ASIN(R1)/ACOS(-1.)
ALAM2=0.5+ASIN(R2)/ACOS(-1.)
P=1.-ALAM1
Q=ALAM1
PLAM=(1.-2.*ALAM1+ALAM2)/(2.*(1.-ALAM1))
V1=(N-1)*P*Q
V2=2.*P*Q*(DLAM-P)/(1.-DLAM)
V3=Q*(1.-((DLAM-P)/Q)**(N-1))/(1.-DLAM)
V4=V2*(N-1-V3)
VD(I+1)=V1+V4
ED(I+1)=(N-1)*P
1 CONTINUE
RETURN
END
EOF AT LINE 41
*

```

## References

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